Maxwell's equations for time-varying fields

Gauss's law $\nabla \cdot \vec{D} = \rho_v$ $\quad \oint_S \vec{D} \cdot d\vec{s} = Q$

Faraday's law $\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$ $\quad \oint_C \vec{E} \cdot d\vec{l} = -\int_S \frac{\partial \vec{B}}{\partial t} \cdot d\vec{s}$

No mag. monopoles $\nabla \cdot \vec{B} = 0$ $\quad \oint_S \vec{B} \cdot d\vec{s} = 0$

Ampere's law $\nabla \times \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t}$ $\quad \oint_C \vec{H} \cdot d\vec{l} = \int_S (\vec{J} + \frac{\partial \vec{D}}{\partial t}) \cdot d\vec{s}$

- Faraday's law $\rightarrow$ <u>changing</u> magnetic fields can produce electric currents (and voltages)
- Induced voltage called <u>electromotive force</u> (emf)

flux through a loop of wire $\Phi = \int_S \vec{B} \cdot d\vec{s}$

$$V_{emf} = -N\frac{d\Phi}{dt} = -N\frac{d}{dt}\int_S \vec{B} \cdot d\vec{s}$$

changing field, stationary loop $\rightarrow$ transformer emf V_{emf}^{tr}

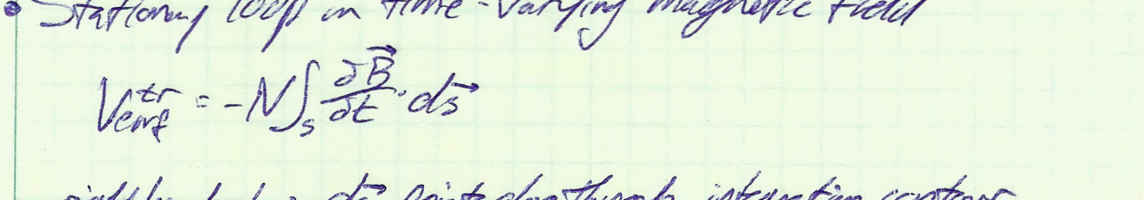
static field, moving loop $\rightarrow$ motional emf V_{emf}^{m}

$$V_{emf} = V_{emf}^{tr} + V_{emf}^{m}$$

- Stationary loop in time-varying magnetic field

$$V_{emf}^{tr} = -N\int_S \frac{\partial \vec{B}}{\partial t} \cdot d\vec{s}$$

right hand rule: $d\vec{s}$ points along thumb, integration contour crosses the gap from positive to negative terminal



$$I = \frac{V_{emf}^{tr}}{R + R_L}$$

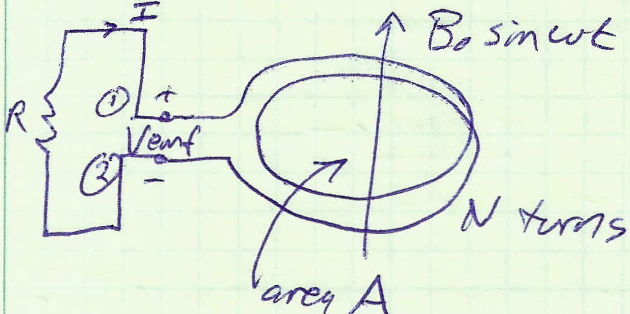
- Lenz's law: Current in loop opposes <u>change</u> in magnetic flux

$$V_{emf}^{tr} = \oint \vec{E} \cdot d\vec{l} = -\int_S \frac{\partial \vec{B}}{\partial t} \cdot d\vec{s}$$

$$\int_S (\nabla \times \vec{E}) \cdot d\vec{s} = -\int_S \frac{\partial \vec{B}}{\partial t} \cdot d\vec{s}$$ stokes' theorem

$$\nabla \times \vec{E} = \frac{-\partial \vec{B}}{\partial t}$$: Faraday's law

Example: coil in changing magnetic field



$$\Phi = \int_S \vec{B} \cdot d\vec{s} = BA = AB_0 \sin \omega t$$

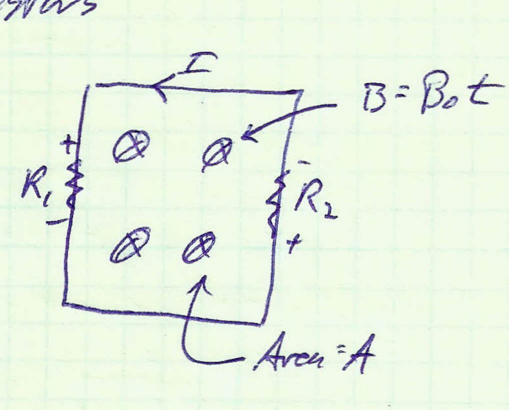
$$V_{emf}^{tr} = -N \frac{\partial \Phi}{\partial t} = -NAB_0 \omega \cos \omega t$$

$V_{emf} = V_1 - V_2$

but $I = \frac{V_2 - V_1}{R} = \frac{-V_{emf}}{R} = \underline{\frac{NAB_0 \omega}{R} \cos \omega t}$

Example: loop of current with resistors

- Note direction of current flow opposes increasing B into page



$V_{emf} = AB_0$

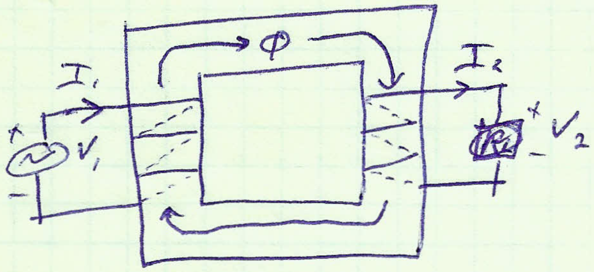
$$I = \frac{V_{emf}}{R_1 + R_2} = \frac{AB_0}{R_1 + R_2}$$

$$V_1 = \frac{AB_0 R_1}{R_1 + R_2} \qquad V_2 = \frac{AB_0 R_2}{R_1 + R_2}$$

The Ideal transformer

$$V_1 = -N_1 \frac{d\phi}{dt}$$

$$V_2 = -N_2 \frac{d\phi}{dt}$$



$$\frac{V_1}{V_2} = \frac{N_1}{N_2}$$

- flux generated by I_2 is opposite to that generated by I_1 (convention for current directions)

if lossless, $P_1 = P_2$

$$V_1 I_1 = V_2 I_2$$

$$\frac{I_1}{I_2} = \frac{N_2}{N_1}$$

Impedance transformation:

$$V_1 = \frac{N_1}{N_2} V_2$$

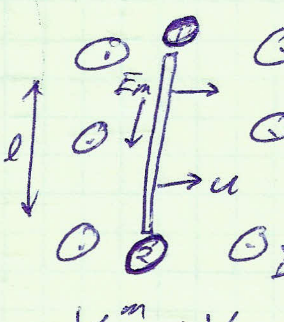
$$I_1 = \frac{N_2}{N_1} I_2$$

$$\rightarrow \frac{V_1}{I_1} = \frac{N_1^2}{N_2^2} \frac{V_2}{I_2}$$

$$Z_{in} = \frac{N_1^2}{N_2^2} Z_L$$

Moving Conductor in Static Magnetic Field

$$F_m = q(\vec{u} \times \vec{B})$$



equivalent electric field $\cancel{E_m} = \frac{F_m}{q} = \vec{u} \times \vec{B}$

called "motional electric field"

$V_{emf}^m = V_{12}$

Motional EMF

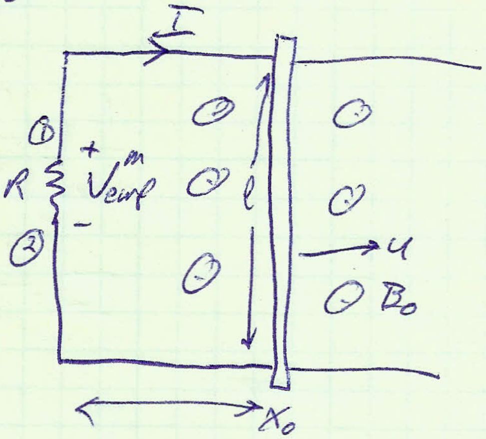
$$V_{emf}^m = \int_2^1 \vec{E}_m \cdot d\vec{l} = \int_2^1 (\vec{u} \times \vec{B}) \cdot d\vec{l}$$

$$\vec{u} \times \vec{B} = \hat{x}u \times \hat{z}B_0$$

$$V_{emf}^m = -uB_0 l$$

for a closed circuit, $V_{emf}^m = \oint_c (\vec{u} \times \vec{B}) \cdot d\vec{l}$

- The classic sliding bar example

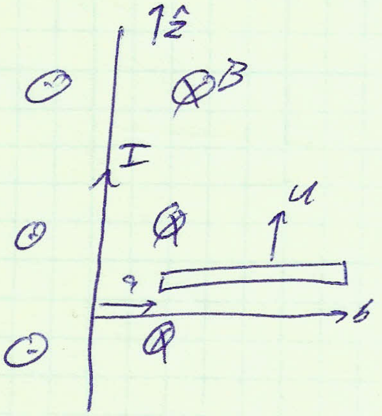


$$\Phi = x_0 B_0 l$$

$$\frac{d\Phi}{dt} = B_0 l \frac{dx_0}{dt} = uB_0 l$$

$$V_{emf}^m = -\frac{d\Phi}{dt} = -uB_0 l$$

- Moving rod next to wire



$$\vec{B} = \hat{\phi}\frac{\mu_0 I}{2\pi r}$$

$$V_{ab} = \int_b^a (\vec{u} \times \vec{B}) \cdot d\vec{l}$$

$$= \int_b^a \left(u\hat{z} \times \hat{\phi}\frac{\mu_0 I}{2\pi r}\right) \cdot \hat{r}\, dr$$

$$= \frac{u\mu_0 I}{2\pi} \ln\left(\frac{a}{b}\right)$$